3.6 The mighty chain rule

used to take the der. of a composite function

composite function: $\quad y = \overset{\text{out}}{f}(\overset{\text{in}}{g}(x))$

function of a function

$$y = f \circ g(x)$$

ex. $y = \sin(x^5)$

out in

$y' = 5x^4 \cos(x^5)$

der of in

der of out (eval at in) ie. not $\cos x$

$$y = \tan(x^2 + 2x + 1)$$

$$y' = (2x+2)\sec^2(x^2+2x+1)$$

---

in

$$y = \sec^3 x = (\sec x)^3$$ out

$$y' = \sec x \tan x \cdot 3(\sec x)^2$$

$$y = (x^2 + 2x + 1)^7$$

inside: x^2+2x+1

outside u^7

der: $7u^6$

$$y' = (2x+2)\,7(x^2+2x+1)^6$$

$u = x^2+2x+1$

$$y = x^2\,(x^2+2x+1)^7$$

product

$$y' = x^2 \cdot \left[(2x+2)\,7(x^2+2x+1)^6\right] + (x^2+2x+1)^7 \cdot 2x$$

p149 chain rule

If $y = f(g(x))$

then $y' = g'(x)\, f'(g(x))$

If $y = f(u)$ $u = g(x)$

out y is a function of u and

in u is a function of x

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

der of inside

der of outside

$y = \sin(x^5)$ u

$$\frac{dy}{dx} = 5x^4 \cos(x^5)$$

$\frac{du}{dx}$ $\frac{dy}{du}$

$$y = f(u) = u^3 + 1 \quad , \quad u = \sin x$$

$$f'(x) \overset{?}{=}$$

$$f(x) = \sin^3 x + 1$$

$$f(x) = (\sin x)^3 + 1$$

$$f'(x) = \cos x \cdot 3(\sin x)^2 + 0$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 3u^2 \cdot \cos x$$

$$= 3(\sin x)^2 \cos x$$